On the calculation of Misiurewicz patterns in one-dimensional quadratic maps

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In this work we give for the first time a table with all Misiurewicz points $\mathrm{M}_{n,p}$ for low values of the preperiod and period $(2 \le n \le 8,\ 1 \le p \le 5)$ in one-dimensional quadratic maps. In the particular case of $\mathrm{M}_{n,1}$ (important Misiurewicz points which are all placed in the period-1 chaotic band) the preperiod values are $(2 \le n \le 11)$. A brute-force algorithm to obtain all the symbolic sequences (patterns) of the $\mathrm{M}_{n,p}$ and a more efficient algorithm to obtain the patterns of $\mathrm{M}_{n,1}$ is also shown.

1. Introduction

Recently, we have used the real axis neighbourhood (antenna) of the Mandelbrot set to study the ordering of hyperbolic components of one-dimensional quadratic maps [1]. This was possible on account of our own graphical tools [2]. For such an ordering different levels of *separators* were required. Separators were Misiurewicz points, and we have studied these points in another paper [3]. In the same way as in these former works, we shall use here the antenna of the Mandelbrot set to find Misiurewicz patterns in one-dimensional quadratic maps.

Misiurewicz points take their name from the mathematician Michal Misiurewicz [4], who became well known in the study of dynamics of one-dimensional maps. These points, which are preperiodic and eventually periodic, were widely studied [4-8]. Unstable or repellent points, which belong to what is sometimes called *the set of* *exceptional points* [9], were known early on. Later, band-merging points [10] which belong to the set of exceptional points were determined.

Misiurewicz points have normally been studied from a mathematical point of view. We tried to do a complementary study of the Misiurewicz points [3] in a more experimental physical way (to the effect that we measured preperiods and periods, we ordered points, we analysed graphics, we obtained rules and properties, etc.). All this work provides a wide view of these points and is closely connected with this article. For instance, in [3] we were forced to introduce new nomenclature because there was a lack terminology, and here we shall use all we introduced there.

As seen in [3], we named Misiurewicz points as $\mathrm{M}_{n,p}$, where n is the preperiod and p the period. Likewise, we represented the symbolic sequence of Misiurewicz points as a sequence of n letters -preperiod- in brackets followed by p letters -period- without brackets. So, as is well known, the band-merging point that separates the period-1 chaotic band and the period-2 chaotic band corresponding to $c = -1.543689012...$ is a Misiurewicz point with $n = 3$ and $p = 1$. Therefore, we name it as $\mathrm{M}_{3,1}$ and we represent its symbolic sequence as (CLR)L.

The number of period-p hyperbolic components of both the Mandelbrot set and its real part has been studied [9, 11-14]. In a similar way, we use here an algorithm to find all the Misiurewicz points $\mathrm{M}_{n,p}$ in the real case, which corresponds to one-dimensional quadratic maps. To do that, we use a brute-force algorithm that allows us to find them even though in a laborious way. As a result of the application of this algorithm, we give for the first time a table with all the Misiurewicz points $\mathrm{M}_{n,p}$ $(2 \le n \le 8,\ 1 \le p \le 5)$ and $\mathrm{M}_{n,1}$ $(2 \le n \le 11)$ of the map $x_{n+1} = x_n^2 + c$, properly ordered. Just like the MSS work [9] (where a table with the complete ordered set of hyperbolic components for $p \le 11$ was given for the first time) was the beginning of a process that culminated in finding formulas to calculate the number of hyperbolic components both in the Manbelbrot set and the Mandelbrot set antenna (even though in most of the cases the logistic map was used), this work is the beginning of a process that will finish with the finding analytic formulas to obtain the number of Misiurewicz points, for a n and p given, in both complex and real cases.

2. Extensions of some MSS concepts

In accordance with the pioneer work of MSS [9], a particular iterate x_n of the map $x_{n+1} = x_n^2 + c$ will be said to be of type L or of type R according as $x_n < 0$ or $x_n > 0$, respectively. Given an initial point x_0, the minimum distinguishing information about the sequence of iterates, will consist in a pattern of R's and L's, the n-th letter giving the relative position of the n-th iterate of x_0 with respect to the critical point $x = 0$. In accordance with this convention, the initial point does not belong to the pattern, and a pattern with $p-1$ letters R or L is said to be of length p. For example, RL is the pattern of the superstable period-3 orbit of the logistic map. This convention has been followed in most cases.

However, a pattern of length p with $p-1$ letters can be misleading. Zheng and Hao [15] write the "symbolic sequence" of a superstable period-p orbit with p letters. For example, the superstable period-3 orbit of a logistic-like map is written as RLC (they assign the letter C when the iterate falls at the critical point). Then, the first letter corresponds to the first iterate, and the last letter corresponds to the initial point (the critical point). Schroeder [16] also writes the "symbolic dynamics" for a given superstable period-p orbit with p letters. For example, CRL for the superstable period-3 orbit of the logistic map. Then, the first letter corresponds to the initial point, the second one to the first iterate, the third one to the second iterate, and so on. We normally use this last procedure to write symbolic sequences (patterns).

There are two types of one-dimensional quadratic maps: logistic-type and Mandelbrot-type. In the case of different type of one-dimensional quadratic maps, the symbolic sequence of an element is obtained from the symbolic sequence of the same element of the other type by interchanging the L's and R's. We can see that in the MSS [9] and in our works L's and R's are interchanged because MSS use the logistic map $x_{n+1} = \lambda x_n(1 - x_n)$ and we use the Mandelbrot real map $x_{n+1} = x_n^2 + c$.

To begin, we shall give three definitions that we shall use to find the Misiurewicz patterns. Normally, we take MSS concepts (or other concepts used by other authors in literature) and we adapt or change them according to our necessities.

Definition 1.

The pattern *P* has even "L-parity" if it has an even number of L's, and has odd "L-parity" otherwise.

Definition 2.

Let *P* be the pattern of a cardioid. The first F-harmonic [1] $H_{\mathrm{F}}^{(1)}(P)$ of *P* is formed by appending *P* to itself and changing the second C to R (or L) if L-parity of *P* is odd (or even). The second F-harmonic $H_{\mathrm{F}}^{(2)}(P)$ of *P* is formed by appending *P* to $H_{\mathrm{F}}^{(1)}(P)$ and changing the second C to R (or L) if L-parity of $H_{\mathrm{F}}^{(1)}(P)$ is odd (or even), and so on .

In this definition, we have changed the Schroeder rule [16] for the calculation of the patterns of MSS-harmonics in order to calculate the patterns of F-harmonics. The periods of the F-harmonics of a pattern are obtained by multiplying the period of the pattern by all the natural numbers, whereas the periods of the MSS-harmonics of a pattern are obtained by multiplying the period of the pattern only by the successive powers of two. These two different types of harmonics of a cardioid have a different physical sense: a MSS-harmonic is a period-doubling cascade disc, whereas a F-harmonic is a last appearance cardioid (with the exception of the first F-harmonic which is a disc). For example, let CLR be the pattern of the period-3 orbit. As is well known, the two first MSS-harmonics [9] of this pattern are $H_{\mathrm{MSS}}^{(1)}(\mathrm{CLR}) = \mathrm{CLR^2LR}$ (a disc) and $H_{\mathrm{MSS}}^{(2)}(\mathrm{CLR}) = \mathrm{CLR^2LRL^2R^2LR}$ (another disc). The three first F-harmonics of the same pattern are $H_{\mathrm{F}}^{(1)}(\mathrm{CLR}) = \mathrm{CLR^2LR}$ (a disc), $H_{\mathrm{F}}^{(2)}(\mathrm{CLR}) = \mathrm{CLR^2LRL^2R}$ (a cardioid) and

$H_{\mathrm{F}}^{(3)}(\mathrm{CLR}) = \mathrm{CLR^2LRL^2RL^2R}$ (another cardioid). Note that $H_{\mathrm{MSS}}^{(1)} = H_{\mathrm{F}}^{(1)}$, but $H_{\mathrm{MSS}}^{(2)} \neq H_{\mathrm{F}}^{(3)}$, though they have the same period. That is so because, for example, $H_{\mathrm{MSS}}^{(2)}(\mathrm{CLR})$ is the period-12 disc that is the second in the period-doubling cascade of the midget CLR, whereas $H_{\mathrm{F}}^{(3)}(\mathrm{CLR})$ is the last appearance period-12 cardioid in the chaotic region of this midget.

Definition 3.

Let P be the pattern of a cardioid. The first F-antiharmonic $A_{\mathrm{F}}^{(1)}(P)$ of P is formed by appending P to itself and changing the second C to L (or R) if L-parity of P is odd (or even). The second F-antiharmonic $A_{\mathrm{F}}^{(2)}(P)$ of P is formed by appending P to $A_{\mathrm{F}}^{(1)}(P)$ and changing the second C to L (or R) if L-parity of $A_{\mathrm{F}}^{(1)}(P)$ is odd (or even), and so on. For example, the two first MSS-antiharmonics [9] of CLR are $A_{\mathrm{MSS}}^{(1)}(\mathrm{CLR}) = \mathrm{CLRL^2R}$ and $A_{\mathrm{MSS}}^{(2)}(\mathrm{CLR}) = \mathrm{CLRL^2RL^2RL^2R}$, and the three first F-antiharmonics of CLR are $A_{\mathrm{F}}^{(1)}(\mathrm{CLR}) = \mathrm{CLRL^2R}$, $A_{\mathrm{F}}^{(2)}(\mathrm{CLR}) = \mathrm{CLRL^2RL^2R}$ and $A_{\mathrm{F}}^{(3)}(\mathrm{CLR}) = \mathrm{CLRL^2RL^2RL^2R}$. Just like a MSS-antiharmonic is a purely formal construct [9] and never corresponds to a periodic orbit, a F-antiharmonic is also a purely formal construct and never corresponds to a periodic orbit. Note that a MSS-antiharmonic coincides with the F-antiharmonic of the same period. For example, $A_{\mathrm{MSS}}^{(2)}(\mathrm{CLR}) = A_{\mathrm{F}}^{(3)}(\mathrm{CLR})$.

The inverse path of a Misiurewicz point must be a l.i.p.

As we can easily see, the symbolic sequence of a period-1 Misiurewicz point $\mathrm{M}_{n,1}$ of the map $x_{n+1} = x_n^2 + c$ has the form $(\mathrm{CL} \ldots \mathrm{R}^{\alpha})\mathrm{L}$ $(\alpha = 1,2,3,\ldots)$, i.e. the symbolic sequence of the periodic part of $\mathrm{M}_{n,1}$ always is L, and the symbolic sequence of the preperiodic part of $\mathrm{M}_{n,1}$ always finishes in a power of R. There is only an exception: $\mathrm{M}_{2,1}$ is $(\mathrm{CL})\mathrm{R}$. Let us designate the x value of the left-hand unstable fixed point by the letter O. If we take away the letter C from the former symbolic sequence and change the last L by O, we have the following symbolic sequence: $\mathrm{L} \ldots \mathrm{R}^{\alpha}\mathrm{O}$. If we read it from right to left $(\mathrm{OR}^{\alpha} \ldots \mathrm{L})$, we have the inverse path [9] of the former period-1

Misiurewicz point. The numerical value of the last iterate of this inverse path is the coordinate [9] of the inverse path. If the pattern actually corresponds to a Misiurewicz point, the coordinate has an absolute value greater than any absolute value of the other iterates of the complete symbolic sequence of $M_{n,1}$, and L ... $R^{\alpha}O$ (read as OR^{α} ... L) is then a legal inverse path (l.i.p.) [9]. For example, let $(CLR^2L^2R)L$ and $(CLRL^3R)L$ be two hypothetical period-1 Misiurewicz points (see fig. 1a and 1b). LR^2L^2RO is a l.i.p, but LRL^3RO is not a l.i.p. In fact, the first pattern is $M_{7,1}$, but the second one is not a Misiurewicz point.

Let $(CLR^{\alpha_1}L^{\beta_1} \ldots R^{\alpha_k}L^{\beta_k})R^{\gamma_1}L^{\delta_1} \ldots R^{\gamma_l}L^{\delta_l}$ be the symbolic sequence of a Misiurewicz point $M_{n,p}$, where $n = 2 + \sum(\alpha_i + \beta_i)$, $i = 1,2,\ldots,k$ and $p = \sum(\gamma_j + \delta_j)$, $j = 1,2,\ldots,l$. We have calculated that, for $p > 1$, the number of possible patterns of the periodic part $R^{\gamma_1}L^{\delta_1} \ldots R^{\gamma_l}L^{\delta_l}$ is

$$N(p) = 2^p - \sum_{d_i/p} 2^{p/d_i} + 2(\nu_i - 1), \qquad (1)$$

where we make the summation over the proper prime divisors of *p* (prime divisors of *p* except *p* itself and 1), and ν_i is the number of the proper prime divisors of *p*. So, for example, $N(4) = 12$ and $N(5) = 30$. In fig. 2 we have drawn the patterns of the periodic part of Misiurewicz points $M_{n,p}$ for $1 \le p \le 5$ (the former formula is not valid for $p = 1$). Note that the periodic part of a period-*p* Misiurewicz point pattern has *p* different inputs (in the figure we have marked each input with an arrow). Let us designate the *x* value of the input to the unstable cycle by the letter O. We call the symbolic sequence $LR^{\alpha_1}L^{\beta_1} \ldots R^{\alpha_k}L^{\beta_k}O$, when we read from right to left, the inverse path of the period-*p* Misiurewicz point. The numerical value of the last iterate of this inverse path is the coordinate of the inverse path. If the pattern actually corresponds to a Misiurewicz point, the coordinate has an absolute value greater that any absolute value of the iterates of the complete sequence of the Misiurewicz point, and then $LR^{\alpha_1}L^{\beta_1} \ldots R^{\alpha_k}L^{\beta_k}O$ is a l.i.p. For example, let $(CLR^2L)L^2R$ and $(CLRL^2)L^2R$ be two hypothetical Misiurewicz points with the same pattern of periodic part (see fig. 1c and 1d). The cycle input is the number 6 (fig. 2c). Let us go back to fig. 1. Then, LR^2LO is a l.i.p, but LRL^2O is not a l.i.p. So, the first pattern corresponds to a Misiurewicz point $M_{5,3}$, but the second one is not a Misiurewicz point.

From this moment we are in a position to search all the Misiurewicz points $M_{n,p}$ of a one-dimensional quadratic map for given values of n and p. For that, we shall use a brute-force algorithm in such a way that we shall test the whole possible patterns, and only in the case of being a l.i.p. they will be a Misiurewicz point.

3. On the calculation of Misiurewicz patterns

Brute-force algorithm

Let $(\mathrm{CLR}\overset{(n\text{-}3)\text{letters}}{\mathrm{X}\ \ldots\ \mathrm{X}})\overset{p\text{ letters}}{\mathrm{X}\ \ldots\ \mathrm{X}}$ be the symbolic sequence of a Misiurewicz point $M_{n,p}$. The number of possible patterns of the periodic part of $M_{n,p}$ (see fig. 2 for $2 \le p \le 5$) is given by eq. (1). Suppose that we have drawn all these patterns. The number of possible patterns of the preperiodic part is 2^{n-3}. But, as we can see in fig. 2, if the periodic part finishes in R the preperiodic part must finish in L and vice versa. Therefore, the possible number of Misiurewicz points $M_{n,p}$ is

$$N(n,p) = 2^{n-4}\left[2^p - \sum_{d_i/p} 2^{p/d_i} + 2(\nu_i - 1)\right]. \tag{2}$$

Starting from this, we test the inverse paths of all the possible patterns which are candidates to be a Misiurewicz point. If the pattern tested is a l.i.p., then it is a true Misiurewicz point, and, if it is not a l.i.p., it is not a Misiurewicz point.

The former algorithm is what we call the brute-force algorithm. By using this algorithm, we have found all the Misiurewicz points for $2 \le n \le 8$ and $1 \le p \le 5$. In the appendix we show table 1, where all these Misiurewicz points are included. The first column shows the Misiurewicz points, where the superindex in brackets indicates the ordering of a given $M_{n,p}$ when the parameter absolute value increases. The second column shows the symbolic sequence (pattern) of the Misiurewicz point, and the third column the parameter value. This table is valid for all the one-dimensional quadratic maps if we take into account same considerations: firstly, in the case of logistic-type one-dimensional quadratic maps we must interchange L and R in patterns, and secondly, the parameter value must be transformed according to a same transformation for every one-dimensional quadratic map (for instance, for the logistic map case the transformation is $\lambda = 1 + \sqrt{1-4c}$).

In table 2 the number of Misiurewicz points for the values $0 \le n \le 8$ and $1 \le p \le 5$ is given, starting from data of the table 1. So, we have 19 Misiurewicz points $M_{7,3}$ and 136 Misiurewicz points $M_{8,5}$. Note that there is no $M_{1,p}$ Misiurewicz point, and that $M_{0,p}$ are the hyperbolic components of period-*p* of MSS.

Table 2
Number of Misiurewicz points $M_{n,p}$ for the values $0 \le n \le 8$ and $1 \le p \le 5$.

p	*n:* 0	1	2	3	4	5	6	7	8
1	1	0	1	1	1	1	3	3	9
2	1	0	0	1	2	3	4	9	14
3	1	0	0	2	3	6	10	19	34
4	2	0	0	4	6	8	18	30	56
5	3	0	0	8	14	20	39	72	136

Let us see the efficiency of the brute-force algorithm. For example, the number of tests used in the case of $M_{7,5}$ was $2^3 \cdot 30 = 240$. In table 2 we can see that the number of $M_{7,5}$ is 72. Therefore, in this case the efficiency of the brute-force algorithm is 30%.

Table 1 is indeed very useful because we have for the first time a complete list of Misiurewicz points $M_{n,p}$ for the Mandelbrot real map, for the values $2 \le n \le 8$, $1 \le p \le 5$ (and $M_{n,1}$ for $2 \le n \le 11$), with their patterns and parameter values that can be used for any one-dimensional quadratic map. There are other approaches to solve this problem in particular cases. For instance, we shall show here an algorithm to find the two Misiurewicz points which limit a tree.

Algorithm to find $M_{n,1}$ Misiurewicz patterns

As we said before, we have used the antenna of the Mandelbrot set to study the ordering of hyperbolic components in one-dimensional quadratic maps. For such an ordering different levels of separators were required [1]. Separators were Misiurewicz points in all cases. The separators we found in the period-1 chaotic band B_0 were always of the form $M_{n,1}$ [1, 3]. Since $M_{n,1}$ are the separators placed on the left and on the right of a tree (generated by a midget) of B_0, we begin by taking into account the generating midget of B_0 that is the "origin" of the $M_{n,1}$ which pattern we want to calculate. The procedure is the following.

1. If the L-parity of the cardioid of the generating midget is even, we obtain the preperiod pattern of the left separator by removing the last L^{α} (if finishes in L^{α}), and we obtain the preperiod pattern of the right separator by adding a R.

If the L-parity of the cardioid of the generating midget is odd, we obtain the preperiod pattern of the left separator by adding a R, and we obtain the preperiod pattern of the right separator by removing the last L^{α} (if finishes in L^{α}).

2. The period pattern of every separator is always L.

3. A separator only actually exists (and therefore is a true Misiurewicz point) if its inverse path is a l.i.p.

The lower part of fig. 3 shows two portions of the antenna of the Mandelbrot set ($-1.975 \leq c \leq -1.969$ for fig. 3a and $-1.864 \leq c \leq -1.840$ for fig. 3b). The upper part of the fig. 3 shows a sketch of the lower part where the periods and patterns of some midgets (and preperiods, periods and patterns of some Misiurewicz points) are indicated. As we can see in fig. 3a, the separators of the tree generated by the midget CLR^3L^3 (this pattern can also be considered the pattern of the midget cardioid) are $(CLR^3)L$ (left) and $(CLR^3L^3R)L$ (right). The inverse path LR^3O is a l.i.p. and therefore

$(CLR^3)L$ is a true Misiurewicz point, $M_{5,1}$. The inverse path LR^3L^3RO is also a l.i.p. and therefore $(CLR^3L^3R)L$ is also a true Misiurewicz point, $M_{9,1}$. Note that in all the cases the two separators generated by a midget have different periods. However, it is possible that one of the two does not exist. For instance, as we can see in fig. 3b, the midget CLR^2LRLR only has the right separator $(CLR^2LRLR)L$, because the left inverse path LR^2LRLR^2O is not a l.i.p. Likewise, the midget 11a only has one separator (left) and the midget 11b another one separator (right).

In fig. 4 the tree generated by the period-3 midget CLR is shown. Only external branches of the new trees generated by the Fourier-harmonics (full circles) and the Fourier-antiharmonics (circles) of the period-3 midget are drawn. The separators (Misiurewicz points) of trees are indicated. As can be seen, a F-harmonic generates a semitree which is situated to the left of the generating tree trunk. The left separator of this semitree can be found by using the former algorithm. So, the left-hand side separator of $H_F^{(2)}(CLR)$ is $(CLR^2LRL^2R)L$, a Misiurewicz point $M_{9,1}$. The corresponding F-antiharmonic generates a semitree situated to the right of the generating tree trunk. So, the right-hand side separator of $A_F^{(2)}(CLR)$ is $(CLRL^2RL^2R)L$, a Misiurewicz point $M_{9,1}$. Note that separators of trees constituted by the two semitrees generated by a pair {F-harmonic, F-antiharmonic} always have the same preperiod. However, these separators can be considered as separators of the period-11 and period-8 trees.

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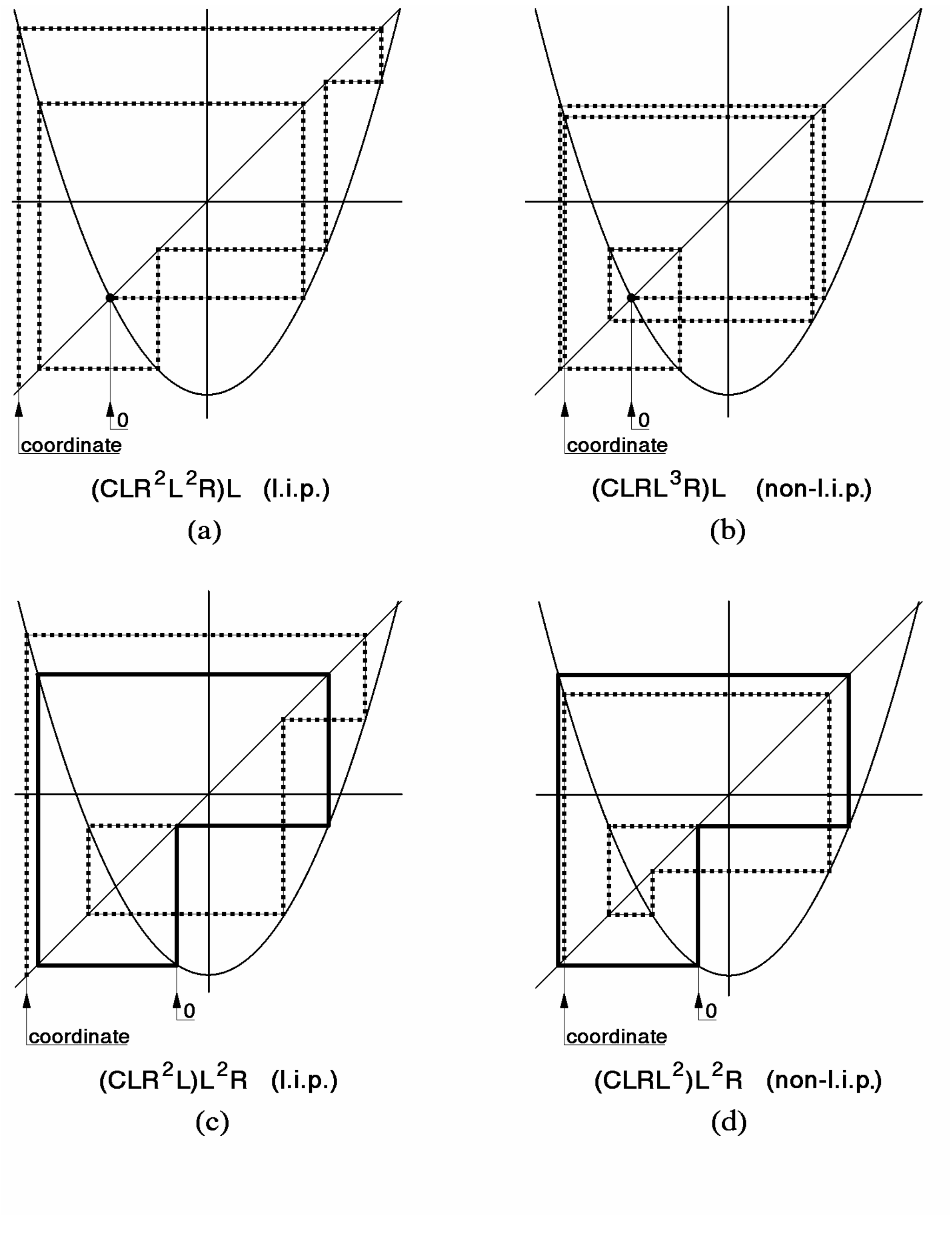
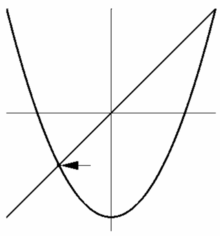
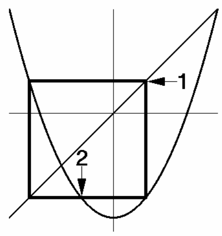


Fig. 1. Four examples of the MSS l.i.p. concept extended to Misiurewicz points.
a) $M_{7,1}$ l.i.p. case, b) non-l.i.p. case for $n = 7$, $p = 1$, c) $M_{5,3}$ l.i.p. case, d) non-l.i.p. case for $n = 5$, $p = 3$.

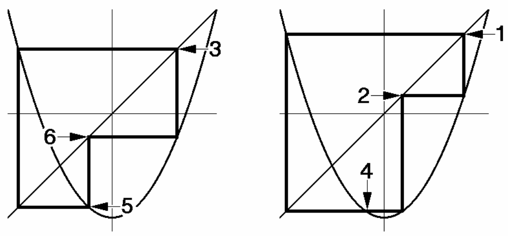


Pattern of $M_{n,1}$
(CL ... R^{α})L



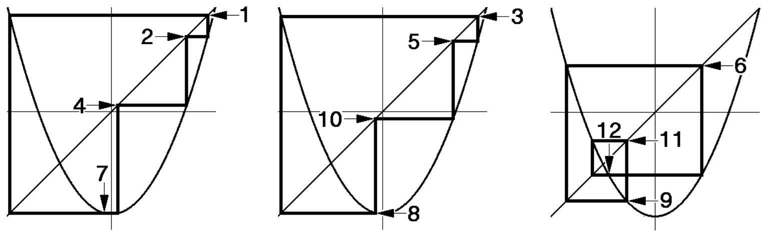
Patterns of $M_{n,2}$	
1	(CL ... R^{α})RL
2	(CL ... L^{β})LR

(a) (b)



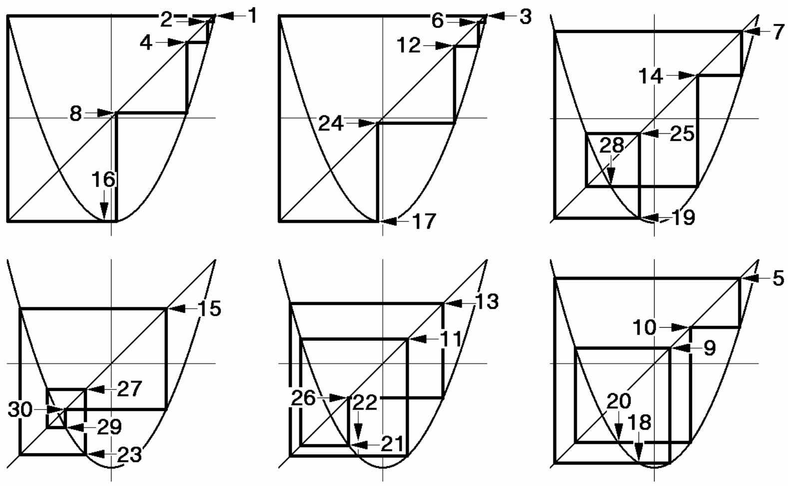
Patterns of $M_{n,3}$			
1	(CL ... R^{α})RRL	4	(CL ... L^{β})LRR
2	(CL ... L^{β})RLR	5	(CL ... R^{α})LRL
3	(CL ... R^{α})RLL	6	(CL ... L^{β})LLR

(c)



Patterns of $M_{n,4}$			
1	(CL ... R^{α})RRRL	7	(CL ... L^{β})LRRR
2	(CL ... L^{β})RRLR	8	(CL ... R^{α})LRRL
3	(CL ... R^{α})RRLL	9	(CL ... R^{α})LRLL
4	(CL ... L^{β})RLRR	10	(CL ... L^{β})LLRR
5	(CL ... L^{β})RLLR	11	(CL ... R^{α})LLRL
6	(CL ... R^{α})RLLL	12	(CL ... L^{β})LLLR

(d)



Patterns of $M_{n,5}$			
1	(CL ... R^{α})RRRRL	16	(CL ... L^{β})LRRRR
2	(CL ... L^{β})RRRLR	17	(CL ... R^{α})LRRRL
3	(CL ... R^{α})RRRLL	18	(CL ... L^{β})LRRLR
4	(CL ... L^{β})RRLRR	19	(CL ... R^{α})LRRLL
5	(CL ... R^{α})RRLRL	20	(CL ... L^{β})LRLRR
6	(CL ... L^{β})RRLLR	21	(CL ... R^{α})LRLRL
7	(CL ... R^{α})RRLLL	22	(CL ... L^{β})LRLLR
8	(CL ... L^{β})RLRRR	23	(CL ... R^{α})LRLLL
9	(CL ... R^{α})RLRRL	24	(CL ... L^{β})LLRRR
10	(CL ... L^{β})RLRLR	25	(CL ... R^{α})LLRRL
11	(CL ... R^{α})RLRLL	26	(CL ... L^{β})LLRLR
12	(CL ... L^{β})RLLRR	27	(CL ... R^{α})LLRLL
13	(CL ... R^{α})RLLRL	28	(CL ... L^{β})LLLRR
14	(CL ... L^{β})RLLLR	29	(CL ... R^{α})LLLRL
15	(CL ... R^{α})RLLLL	30	(CL ... L^{β})LLLLR

(e)

Fig. 2. Patterns of the periodic part of Misiurewicz points $M_{n,p}$ for $1 \le p \le 5$.

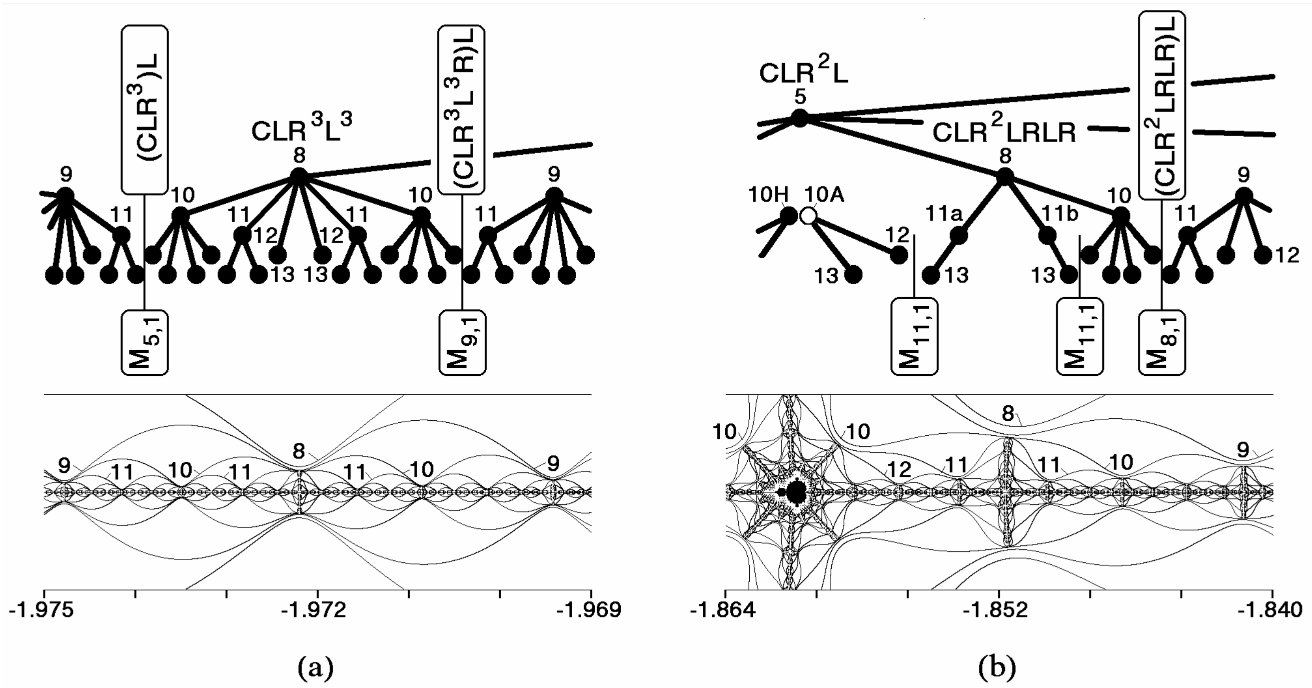


Fig. 3. Examples of calculation of separators $M_{n,1}$. a) Two separators case. b) One separator case.

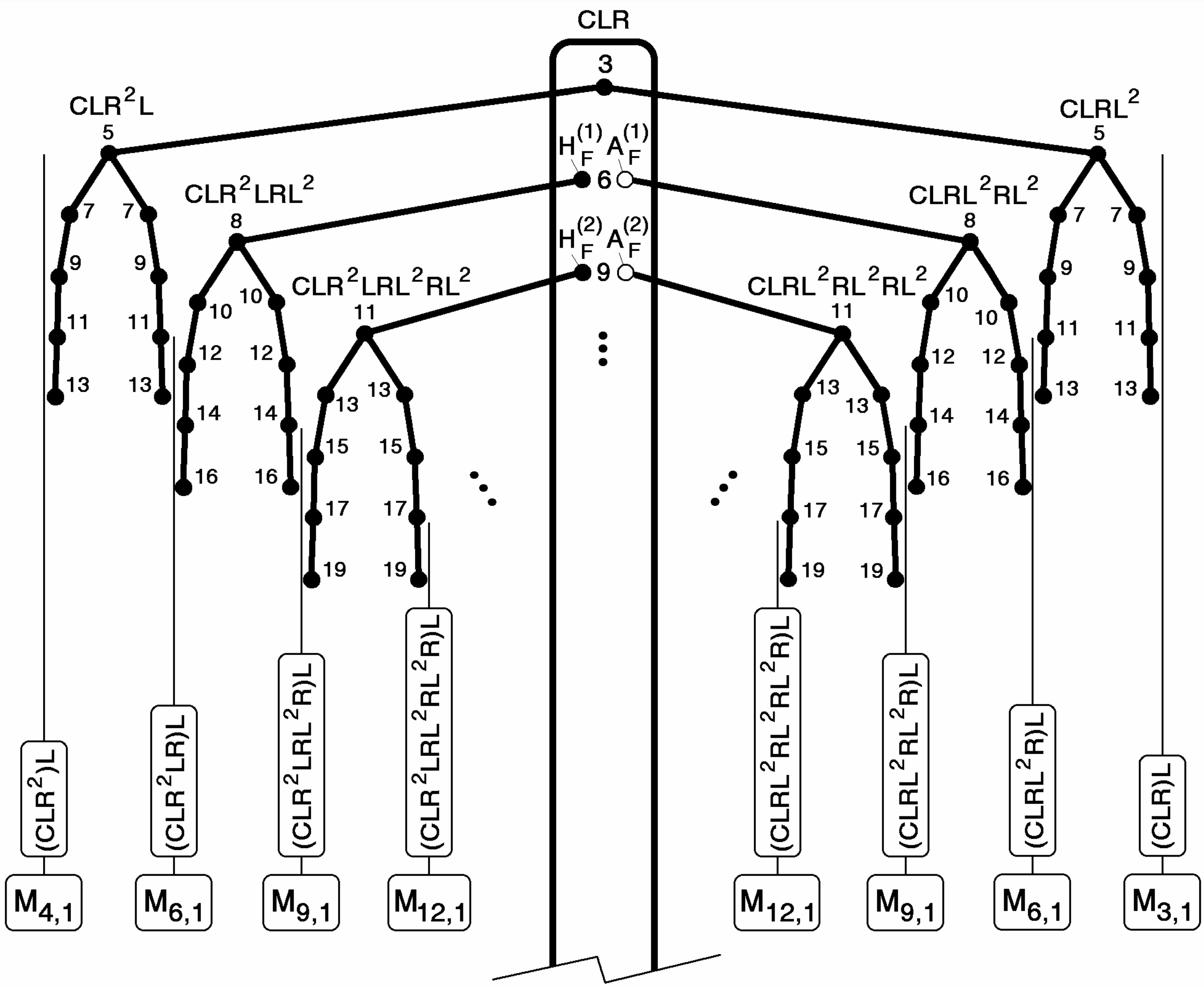


Fig. 4. Separators of trees constituted by the two semitrees generated by the pair {F-harmonic, F-antiharmonic}.

Appendix

Table I
Misiurewicz points $M_{n,p}$ $(2 \le n \le 8,\ 1 \le p \le 5)$ and $M_{n,1}$ $(2 \le n \le 11)$ of the map $x_{n+1} = x_n^2 + c$.

$M^{(1)}_{5,2}$	$(CLRL^2)LR$	-1.430357632...	$M^{(6)}_{8,5}$	$(CLRL^2RL^2)L^2RLR$	-1.702618945...
$M^{(1)}_{7,4}$	$(CLRL^3R)LRL^2$	-1.454820744...	$M^{(3)}_{7,4}$	$(CLRL^2RL)L^3R$	-1.705500281...
$M^{(1)}_{5,4}$	$(CLRL^2)L^3R$	-1.496464687...	$M^{(1)}_{11,1}$	$(CLRL^2RL^4R)L$	-1.706548446...
$M^{(1)}_{7,2}$	$(CLRL^4)LR$	-1.509171750...	$M^{(1)}_{3,5}$	$(CLR)L^2RL^2$	-1.707364499...
$M^{(2)}_{7,4}$	$(CLRL^4)L^3R$	-1.528569621...	$M^{(1)}_{4,5}$	$(CLRL)LRL^2R$	-1.714413091...
$M^{(1)}_{3,1}$	$(CLR)L$	-1.543689012...	$M^{(2)}_{11,1}$	$(CLRL^2RL^2RLR)L$	-1.716020359...
$M^{(1)}_{8,4}$	$(CLRL^5)L^3R$	-1.551783943...	$M^{(2)}_{7,2}$	$(CLRL^2RL)LR$	-1.718974533...
$M^{(1)}_{8,2}$	$(CLRL^5)LR$	-1.560984645...	$M^{(3)}_{6,4}$	$(CLRL^2R)L^2RL$	-1.724289723...
$M^{(1)}_{6,4}$	$(CLRL^3)L^3R$	-1.565454316...	$M^{(1)}_{9,1}$	$(CLRL^2RL^2R)L$	-1.727954141...
$M^{(1)}_{10,1}$	$(CLRL^6R)L$	-1.570697203...	$M^{(4)}_{6,5}$	$(CLRL^2R)L^2RL^2$	-1.730715221...
$M^{(2)}_{10,1}$	$(CLRL^4RLR)L$	-1.579747189...	$M^{(3)}_{7,5}$	$(CLRL^2RL)LRL^2R$	-1.733218127...
$M^{(2)}_{8,4}$	$(CLRL^4R)LRL^2$	-1.584898604...	$M^{(1)}_{7,3}$	$(CLR^2LRL)LR^2$	-1.782233250...
$M^{(1)}_{6,2}$	$(CLRL^3)LR$	-1.589484400...	$M^{(1)}_{4,3}$	$(CLR^2)LRL$	-1.790327491...
$M^{(1)}_{4,4}$	$(CLRL)L^3R$	-1.599998557...	$M^{(4)}_{7,5}$	$(CLR^2LRL)LRL^2R$	-1.795137866...
$M^{(1)}_{8,1}$	$(CLRL^4R)L$	-1.609479175...	$M^{(5)}_{6,5}$	$(CLR^2LR)L^2RL^2$	-1.797005542...
$M^{(1)}_{6,5}$	$(CLRL^2R)LRL^3$	-1.633358703...	$M^{(2)}_{9,1}$	$(CLR^2LRL^2R)L$	-1.798553464...
$M^{(2)}_{8,1}$	$(CLRL^2RLR)L$	-1.644318465...	$M^{(4)}_{6,4}$	$(CLR^2LR)L^2RL$	-1.800769328...
$M^{(2)}_{6,4}$	$(CLRL^2R)LRL^2$	-1.652190877...	$M^{(3)}_{7,2}$	$(CLR^2LRL)LR$	-1.804369886...
$M^{(1)}_{8,5}$	$(CLRL^2RLR)L^3RL$	-1.654315947...	$M^{(3)}_{11,1}$	$(CLR^2LRL^2RLR)L$	-1.805983276...
$M^{(2)}_{8,5}$	$(CLRL^2RLR)LRLRL$	-1.657460990...	$M^{(2)}_{3,5}$	$(CLR)RLRL^2$	-1.806758392...
$M^{(1)}_{4,2}$	$(CLRL)LR$	-1.661239227...	$M^{(1)}_{8,3}$	$(CLR^2LRL^2)RLR$	-1.808646299...
$M^{(3)}_{8,4}$	$(CLRL^2RLR)LRL^2$	-1.664496938...	$M^{(2)}_{8,3}$	$(CLR^2LRL^2)L^2R$	-1.812356027...
$M^{(3)}_{10,1}$	$(CLRL^2RLRLR)L$	-1.666910817...	$M^{(2)}_{4,5}$	$(CLR^2)LRL^3$	-1.813384361...
$M^{(3)}_{8,5}$	$(CLRL^2RLR)LRL^3$	-1.668989710...	$M^{(4)}_{11,1}$	$(CLR^2LRL^4R)L$	-1.813673972...
$M^{(2)}_{6,5}$	$(CLRL^2R)LRLRL$	-1.670206829...	$M^{(4)}_{7,4}$	$(CLR^2LRL)L^3R$	-1.814120900...
$M^{(4)}_{8,5}$	$(CLRL^2RL^2)LRL^2R$	-1.678187715...	$M^{(7)}_{8,5}$	$(CLR^2LRL^2)L^2RLR$	-1.815472060...
$M^{(3)}_{6,5}$	$(CLRL^2R)L^3RL$	-1.679420896...	$M^{(8)}_{8,5}$	$(CLR^2LRL^2)L^4R$	-1.817183956...
$M^{(4)}_{10,1}$	$(CLRL^2RL^3R)L$	-1.681245227...	$M^{(2)}_{6,1}$	$(CLR^2LR)L$	-1.818620134...
$M^{(1)}_{3,4}$	$(CLR)L^2RL$	-1.683316983...	$M^{(5)}_{8,4}$	$(CLR^2LRL^2)L^3R$	-1.820877948...
$M^{(2)}_{8,2}$	$(CLRL^2RL^2)LR$	-1.686061200...	$M^{(5)}_{7,5}$	$(CLR^2LRL)L^4R$	-1.821213808...
$M^{(1)}_{7,5}$	$(CLRL^2RL)L^2RLR$	-1.688666865...	$M^{(6)}_{7,5}$	$(CLR^2LRL)L^2RLR$	-1.824147573...
$M^{(2)}_{7,5}$	$(CLRL^2RL)L^4R$	-1.691985575...	$M^{(3)}_{8,2}$	$(CLR^2LRL^2)LR$	-1.825016794...
$M^{(4)}_{8,4}$	$(CLRL^2RL^2)L^3R$	-1.693112051...	$M^{(2)}_{4,4}$	$(CLR^2)LRL^2$	-1.826254739...
$M^{(1)}_{6,1}$	$(CLRL^2R)L$	-1.697555393...	$M^{(5)}_{10,1}$	$(CLR^2LRL^3R)L$	-1.826912689...
$M^{(5)}_{8,5}$	$(CLRL^2RL^2)L^4R$	-1.700617983...	$M^{(6)}_{6,5}$	$(CLR^2LR)L^3RL$	-1.827323981...

$M^{(9)}_{8,5}$	$(CLR^2LRL^2)LRL^2R$	-1.827831761...
$M^{(2)}_{7,3}$	$(CLR^2LRL)L^2R$	-1.828702096...
$M^{(3)}_{8,3}$	$(CLR^2LRL^2)LR^2$	-1.830091471...
$M^{(3)}_{7,3}$	$(CLR^2LRL)RLR$	-1.834579155...
$M^{(4)}_{8,3}$	$(CLR^2LRLR)LRL$	-1.835948251...
$M^{(7)}_{6,5}$	$(CLR^2LR)LRLRL$	-1.836740247...
$M^{(10)}_{8,5}$	$(CLR^2LRLR)LRL^3$	-1.837215521...
$M^{(6)}_{10,1}$	$(CLR^2LRLRLR)L$	-1.837585735...
$M^{(6)}_{8,4}$	$(CLR^2LRLR)LRL^2$	-1.838176059...
$M^{(1)}_{3,2}$	$(CLR)RL$	-1.839286755...
$M^{(11)}_{8,5}$	$(CLR^2LRLR)LRLRL$	-1.840027890...
$M^{(12)}_{8,5}$	$(CLR^2LRLR)L^3RL$	-1.842663462...
$M^{(5)}_{6,4}$	$(CLR^2LR)LRL^2$	-1.842926100...
$M^{(3)}_{8,1}$	$(CLR^2LRLR)L$	-1.844768746...
$M^{(8)}_{6,5}$	$(CLR^2LR)LRL^3$	-1.845884192...
$M^{(3)}_{4,5}$	$(CLR^2)LRLRL$	-1.847319461...
$M^{(7)}_{8,4}$	$(CLR^2LRLR)L^2RL$	-1.848318050...
$M^{(5)}_{11,1}$	$(CLR^2LRLRL^2R)L$	-1.848633388...
$M^{(13)}_{8,5}$	$(CLR^2LRLR)L^2RL^2$	-1.848828602...
$M^{(1)}_{6,3}$	$(CLR^2LR)LRL$	-1.849488539...
$M^{(1)}_{5,3}$	$(CLR^2L)RLR$	-1.853366536...
$M^{(14)}_{8,5}$	$(CLR^2LRLR)RLRL^2$	-1.854840394...
$M^{(6)}_{11,1}$	$(CLR^2LRLR^2LR)L$	-1.855490810...
$M^{(4)}_{8,2}$	$(CLR^2LRLR)RL$	-1.857037971...
$M^{(9)}_{6,5}$	$(CLR^2L^2)LR^2LR$	-1.862331090...
$M^{(5)}_{8,2}$	$(CLR^2L^3R)RL$	-1.865170449...
$M^{(7)}_{11,1}$	$(CLR^2L^3R^2LR)L$	-1.866578727...
$M^{(15)}_{8,5}$	$(CLR^2L^3R)RLRL^2$	-1.867154094...
$M^{(2)}_{6,3}$	$(CLR^2L^2)LR^2$	-1.868429441...
$M^{(16)}_{8,5}$	$(CLR^2L^3R)RLR^2L$	-1.869425053...
$M^{(17)}_{8,5}$	$(CLR^2L^3R)L^2R^2L$	-1.870445699...
$M^{(2)}_{5,3}$	$(CLR^2L)L^2R$	-1.872067943...
$M^{(18)}_{8,5}$	$(CLR^2L^3R)L^2RL^2$	-1.872569270...
$M^{(8)}_{11,1}$	$(CLR^2L^3RL^2R)L$	-1.872717120...
$M^{(8)}_{8,4}$	$(CLR^2L^3R)L^2RL$	-1.872960068...
$M^{(10)}_{6,5}$	$(CLR^2L^2)LRL^2R$	-1.873736044...
$M^{(4)}_{4,5}$	$(CLR^2)L^3RL$	-1.874926621...
$M^{(4)}_{8,1}$	$(CLR^2L^3R)L$	-1.875762032...
$M^{(2)}_{3,4}$	$(CLR)RL^3$	-1.877132157...
$M^{(19)}_{8,5}$	$(CLR^2L^3R)L^3RL$	-1.877306096...
$M^{(7)}_{7,5}$	$(CLR^2L^3)RL^3R$	-1.878147705...
$M^{(8)}_{7,5}$	$(CLR^2L^3)RLRLR$	-1.878678481...
$M^{(20)}_{8,5}$	$(CLR^2L^3R)LRLRL$	-1.879362818...
$M^{(2)}_{6,2}$	$(CLR^2L^2)LR$	-1.879839142...
$M^{(9)}_{8,4}$	$(CLR^2L^3R)LRL^2$	-1.880592897...
$M^{(7)}_{10,1}$	$(CLR^2L^3RLR)L$	-1.880960056...
$M^{(21)}_{8,5}$	$(CLR^2L^3R)LRL^3$	-1.881178176...
$M^{(1)}_{5,5}$	$(CLR^2L)L^2RLR$	-1.881488283...
$M^{(5)}_{8,3}$	$(CLR^2L^3R)LRL$	-1.881895872...
$M^{(4)}_{7,3}$	$(CLR^2L^3)RLR$	-1.882792429...
$M^{(11)}_{6,5}$	$(CLR^2L^2)LRLR^2$	-1.883708116...
$M^{(22)}_{8,5}$	$(CLR^2L^3R)LR^2L^2$	-1.883884650...
$M^{(12)}_{6,5}$	$(CLR^2L^2)L^3R^2$	-1.885751313...
$M^{(23)}_{8,5}$	$(CLR^2L^4)LR^2LR$	-1.885930360...
$M^{(6)}_{8,3}$	$(CLR^2L^4)LR^2$	-1.886801772...
$M^{(5)}_{7,3}$	$(CLR^2L^3)L^2R$	-1.887658620...
$M^{(24)}_{8,5}$	$(CLR^2L^4)LRL^2R$	-1.888028880...
$M^{(2)}_{5,5}$	$(CLR^2L)L^4R$	-1.888314454...
$M^{(8)}_{10,1}$	$(CLR^2L^5R)L$	-1.888510231...
$M^{(6)}_{6,4}$	$(CLR^2L^2)L^3R$	-1.888838375...
$M^{(6)}_{8,2}$	$(CLR^2L^4)LR$	-1.889508462...
$M^{(9)}_{7,5}$	$(CLR^2L^3)L^2RLR$	-1.889916404...
$M^{(25)}_{8,5}$	$(CLR^2L^4)LRLR^2$	-1.890459342...
$M^{(26)}_{8,5}$	$(CLR^2L^4)L^3R^2$	-1.891046803...
$M^{(10)}_{7,5}$	$(CLR^2L^3)L^4R$	-1.891693379...
$M^{(10)}_{8,4}$	$(CLR^2L^4)L^3R$	-1.891829290...
$M^{(1)}_{4,1}$	$(CLR^2)L$	-1.892910987...
$M^{(27)}_{8,5}$	$(CLR^2L^4)L^4R$	-1.893535931...
$M^{(28)}_{8,5}$	$(CLR^2L^4)L^2RLR$	-1.894434616...
$M^{(5)}_{7,4}$	$(CLR^2L^3)L^3R$	-1.894962860...
$M^{(9)}_{11,1}$	$(CLR^2L^6R)L$	-1.895120666...
$M^{(13)}_{6,5}$	$(CLR^2L^2)L^4R$	-1.895213754...
$M^{(7)}_{8,3}$	$(CLR^2L^4)L^2R$	-1.895522478...
$M^{(11)}_{7,5}$	$(CLR^2L^3)L^3R^2$	-1.896376161...
$M^{(12)}_{7,5}$	$(CLR^2L^3)LRLR^2$	-1.897550763...
$M^{(8)}_{8,3}$	$(CLR^2L^4)RLR$	-1.897912271...
$M^{(14)}_{6,5}$	$(CLR^2L^2)L^2RLR$	-1.898490485...
$M^{(10)}_{11,1}$	$(CLR^2L^4RLR)L$	-1.898719317...

$M_{7,2}^{(4)}$	$(CLR^2L^3)LR$	-1.899201463...
$M_{8,5}^{(29)}$	$(CLR^2L^4)RLRLR$	-1.899653988...
$M_{8,5}^{(30)}$	$(CLR^2L^4)RL^3R$	-1.899988521...
$M_{5,4}^{(2)}$	$(CLR^2L)L^3R$	-1.900368322...
$M_{9,1}^{(3)}$	$(CLR^2L^4R)L$	-1.900921074...
$M_{3,5}^{(3)}$	$(CLR)RL^4$	-1.901244200...
$M_{7,5}^{(13)}$	$(CLR^2L^3)LRL^2R$	-1.901725649...
$M_{6,3}^{(3)}$	$(CLR^2L^2)L^2R$	-1.902308553...
$M_{7,3}^{(6)}$	$(CLR^2L^3)LR^2$	-1.903729646...
$M_{7,5}^{(14)}$	$(CLR^2L^3)LR^2LR$	-1.904888474...
$M_{5,5}^{(3)}$	$(CLR^2L)L^3R^2$	-1.905165197...
$M_{10,1}^{(9)}$	$(CLR^2L^4R^2)L$	-1.906042321...
$M_{10,1}^{(10)}$	$(CLR^2L^2RLR^2)L$	-1.908622711...
$M_{7,5}^{(15)}$	$(CLR^2L^2R)LR^2L^2$	-1.909471405...
$M_{5,5}^{(4)}$	$(CLR^2L)LRLR^2$	-1.909751846...
$M_{6,3}^{(4)}$	$(CLR^2L^2)RLR$	-1.910846666...
$M_{7,3}^{(7)}$	$(CLR^2L^2R)LRL$	-1.912223006...
$M_{3,5}^{(4)}$	$(CLR)RL^2RL$	-1.912751647...
$M_{7,5}^{(16)}$	$(CLR^2L^2R)LRL^3$	-1.913200983...
$M_{9,1}^{(4)}$	$(CLR^2L^2RLR)L$	-1.913492335...
$M_{7,4}^{(6)}$	$(CLR^2L^2R)LRL^2$	-1.913990645...
$M_{8,5}^{(31)}$	$(CLR^2L^2RL)RL^3R$	-1.914314495...
$M_{8,5}^{(32)}$	$(CLR^2L^2RL)RLRLR$	-1.914665351...
$M_{5,2}^{(2)}$	$(CLR^2L)LR$	-1.915047212...
$M_{11,1}^{(11)}$	$(CLR^2L^2RLRLR)L$	-1.915466979...
$M_{7,5}^{(17)}$	$(CLR^2L^2R)LRLRL$	-1.915663214...
$M_{8,3}^{(9)}$	$(CLR^2L^2RL)RLR$	-1.916158440...
$M_{6,5}^{(15)}$	$(CLR^2L^2)RLRLR$	-1.916423993...
$M_{6,5}^{(16)}$	$(CLR^2L^2)RL^3R$	-1.917704687...
$M_{8,3}^{(10)}$	$(CLR^2L^2RL)L^2R$	-1.918383061...
$M_{7,5}^{(18)}$	$(CLR^2L^2R)L^3RL$	-1.918629105...
$M_{11,1}^{(12)}$	$(CLR^2L^2RL^3R)L$	-1.918702740...
$M_{4,4}^{(3)}$	$(CLR^2)L^2RL$	-1.918829589...
$M_{8,5}^{(33)}$	$(CLR^2L^2RL)L^2RLR$	-1.919260107...
$M_{8,5}^{(34)}$	$(CLR^2L^2RL)L^4R$	-1.920036836...
$M_{7,1}^{(1)}$	$(CLR^2L^2R)L$	-1.920538736...
$M_{8,4}^{(11)}$	$(CLR^2L^2RL)L^3R$	-1.921410147...
$M_{4,5}^{(5)}$	$(CLR^2)L^2RL^2$	-1.921510772...
$M_{8,5}^{(35)}$	$(CLR^2L^2RL)L^3R^2$	-1.921969418...
$M_{8,5}^{(36)}$	$(CLR^2L^2RL)LRLR^2$	-1.922639645...
$M_{5,5}^{(5)}$	$(CLR^2L)LRL^2R$	-1.923009350...
$M_{8,2}^{(7)}$	$(CLR^2L^2RL)LR$	-1.923309591...
$M_{7,4}^{(7)}$	$(CLR^2L^2R)L^2RL$	-1.923826796...
$M_{10,1}^{(11)}$	$(CLR^2L^2RL^2R)L$	-1.924065307...
$M_{7,5}^{(19)}$	$(CLR^2L^2R)L^2RL^2$	-1.924202854...
$M_{8,5}^{(37)}$	$(CLR^2L^2RL)LRL^2R$	-1.924419058...
$M_{3,3}^{(1)}$	$(CLR)RL^2$	-1.924661063...
$M_{8,3}^{(11)}$	$(CLR^2L^2RL)LR^2$	-1.925320748...
$M_{8,5}^{(38)}$	$(CLR^2L^2RL)LR^2LR$	-1.925778812...
$M_{7,5}^{(20)}$	$(CLR^2L^2R)L^2R^2L$	-1.925908061...
$M_{11,1}^{(13)}$	$(CLR^2L^2RL^2R^2)L$	-1.926202427...
$M_{11,1}^{(14)}$	$(CLR^2L^2R^2LR^2)L$	-1.928126359...
$M_{8,5}^{(39)}$	$(CLR^2L^2R^2)LR^2L^2$	-1.928423507...
$M_{7,5}^{(21)}$	$(CLR^2L^2R)RLR^2L$	-1.928557373...
$M_{5,3}^{(3)}$	$(CLR^2L)LR^2$	-1.929021989...
$M_{8,3}^{(12)}$	$(CLR^2L^2R^2)LRL$	-1.929710512...
$M_{7,5}^{(22)}$	$(CLR^2L^2R)RLRL^2$	-1.929958918...
$M_{8,5}^{(40)}$	$(CLR^2L^2R^2)LRL^3$	-1.930185568...
$M_{10,1}^{(12)}$	$(CLR^2L^2R^2LR)L$	-1.930328287...
$M_{8,4}^{(12)}$	$(CLR^2L^2R^2)LRL^2$	-1.930577396...
$M_{7,2}^{(5)}$	$(CLR^2L^2R)RL$	-1.931126345...
$M_{8,5}^{(41)}$	$(CLR^2L^2R^2)LRLRL$	-1.931443446...
$M_{5,5}^{(6)}$	$(CLR^2L)LR^2LR$	-1.931829828...
$M_{4,5}^{(6)}$	$(CLR^2)L^2R^2L$	-1.932626430...
$M_{8,5}^{(42)}$	$(CLR^2L^2R^2)L^3RL$	-1.933128101...
$M_{7,4}^{(8)}$	$(CLR^2L^2R)RL^3$	-1.933239535...
$M_{8,1}^{(5)}$	$(CLR^2L^2R^2)L$	-1.934258045...
$M_{7,5}^{(23)}$	$(CLR^2L^2R)RL^4$	-1.934866696...
$M_{7,5}^{(24)}$	$(CLR^2L^2R)RL^2RL$	-1.935905848...
$M_{8,4}^{(13)}$	$(CLR^2L^2R^2)L^2RL$	-1.936505761...
$M_{11,1}^{(15)}$	$(CLR^2L^2R^2L^2R)L$	-1.936688243...
$M_{8,5}^{(43)}$	$(CLR^2L^2R^2)L^2RL^2$	-1.936794735...
$M_{7,3}^{(8)}$	$(CLR^2L^2R)RL^2$	-1.937164183...
$M_{8,5}^{(44)}$	$(CLR^2L^2R^2)L^2R^2L$	-1.938327875...
$M_{5,4}^{(3)}$	$(CLR^3)LR^2L$	-1.942762011...
$M_{8,5}^{(45)}$	$(CLR^3LR^2)L^2R^2L$	-1.944224618...
$M_{7,3}^{(9)}$	$(CLR^3LR)RL^2$	-1.945289009...
$M_{8,5}^{(46)}$	$(CLR^3LR^2)L^2RL^2$	-1.945619724...

$M_{11,1}^{(16)}$	$(CLR^3LR^2L^2R)L$	-1.945714205...
$M_{8,4}^{(14)}$	$(CLR^3LR^2)L^2RL$	-1.945876537...
$M_{7,5}^{(25)}$	$(CLR^3LR)RL^2RL$	-1.946409259...
$M_{7,5}^{(26)}$	$(CLR^3LR)RL^4$	-1.947340783...
$M_{8,1}^{(6)}$	$(CLR^3LR^2)L$	-1.947863151...
$M_{7,4}^{(9)}$	$(CLR^3LR)RL^3$	-1.948728057...
$M_{8,5}^{(47)}$	$(CLR^3LR^2)L^3RL$	-1.948818118...
$M_{5,5}^{(7)}$	$(CLR^3)LR^2L^2$	-1.949219735...
$M_{4,5}^{(7)}$	$(CLR^2)RLR^2L$	-1.949954265...
$M_{8,5}^{(48)}$	$(CLR^3LR^2)LRLRL$	-1.950251481...
$M_{7,2}^{(6)}$	$(CLR^3LR)RL$	-1.950499504...
$M_{8,4}^{(15)}$	$(CLR^3LR^2)LRL^2$	-1.950937717...
$M_{10,1}^{(13)}$	$(CLR^3LR^2LR)L$	-1.951129209...
$M_{8,5}^{(49)}$	$(CLR^3LR^2)LRL^3$	-1.951236731...
$M_{7,5}^{(27)}$	$(CLR^3LR)RLRL^2$	-1.951414052...
$M_{8,3}^{(13)}$	$(CLR^3LR^2)LRL$	-1.951593791...
$M_{3,3}^{(2)}$	$(CLR)R^2L$	-1.952133665...
$M_{7,5}^{(28)}$	$(CLR^3LR)RLR^2L$	-1.952441687...
$M_{8,5}^{(50)}$	$(CLR^3LR^2)LR^2L^2$	-1.952542458...
$M_{11,1}^{(17)}$	$(CLR^3LR^2LR^2)L$	-1.952717502...
$M_{8,4}^{(16)}$	$(CLR^3LR^2)LR^2L$	-1.953186986...
$M_{6,4}^{(7)}$	$(CLR^3L)R^2LR$	-1.953312507...
$M_{8,4}^{(17)}$	$(CLR^3LRL)LR^3$	-1.954107987...
$M_{6,4}^{(8)}$	$(CLR^3L)RL^2R$	-1.954233998...
$M_{11,1}^{(18)}$	$(CLR^3LRL^2R^2)L$	-1.954678121...
$M_{7,5}^{(29)}$	$(CLR^3LR)L^2R^2L$	-1.954840291...
$M_{8,5}^{(51)}$	$(CLR^3LRL)LR^2LR$	-1.954934214...
$M_{8,3}^{(14)}$	$(CLR^3LRL)LR^2$	-1.955212714...
$M_{5,3}^{(4)}$	$(CLR^3)LRL$	-1.955694045...
$M_{8,5}^{(52)}$	$(CLR^3LRL)LRL^2R$	-1.955847577...
$M_{7,5}^{(30)}$	$(CLR^3LR)L^2RL^2$	-1.955999036...
$M_{10,1}^{(14)}$	$(CLR^3LRL^2R)L$	-1.956089037...
$M_{7,4}^{(10)}$	$(CLR^3LR)L^2RL$	-1.956247950...
$M_{8,2}^{(8)}$	$(CLR^3LRL)LR$	-1.956605421...
$M_{4,5}^{(8)}$	$(CLR^2)RLRL^2$	-1.956800401...
$M_{8,5}^{(53)}$	$(CLR^3LRL)LRLR^2$	-1.957026792...
$M_{8,5}^{(54)}$	$(CLR^3LRL)L^3R^2$	-1.957598715...
$M_{5,5}^{(8)}$	$(CLR^3)LRL^3$	-1.957878330...
$M_{8,4}^{(18)}$	$(CLR^3LRL)L^3R$	-1.957939385...
$M_{7,1}^{(2)}$	$(CLR^3LR)L$	-1.958509587...
$M_{8,5}^{(55)}$	$(CLR^3LRL)L^4R$	-1.958825915...
$M_{8,5}^{(56)}$	$(CLR^3LRL)L^2RLR$	-1.959351753...
$M_{5,4}^{(4)}$	$(CLR^3)LRL^2$	-1.959612214...
$M_{11,1}^{(19)}$	$(CLR^3LRL^3R)L$	-1.959685873...
$M_{7,5}^{(31)}$	$(CLR^3LR)L^3RL$	-1.959727265...
$M_{8,3}^{(15)}$	$(CLR^3LRL)L^2R$	-1.959866514...
$M_{6,5}^{(17)}$	$(CLR^3L)RL^3R$	-1.960247355...
$M_{8,4}^{(19)}$	$(CLR^3LRL)L^2R^2$	-1.960488699...
$M_{8,4}^{(20)}$	$(CLR^3LRL)RLR^2$	-1.960977585...
$M_{6,5}^{(18)}$	$(CLR^3L)RLRLR$	-1.961309931...
$M_{8,3}^{(16)}$	$(CLR^3LRL)RLR$	-1.961428012...
$M_{7,5}^{(32)}$	$(CLR^3LR)LRLRL$	-1.961708671...
$M_{11,1}^{(20)}$	$(CLR^3LRLRLR)L$	-1.961816119...
$M_{4,2}^{(2)}$	$(CLR^2)RL$	-1.962048398...
$M_{8,5}^{(57)}$	$(CLR^3LRL)RLRLR$	-1.962238190...
$M_{8,5}^{(58)}$	$(CLR^3LRL)RL^3R$	-1.962508497...
$M_{7,4}^{(11)}$	$(CLR^3LR)LRL^2$	-1.962664955...
$M_{9,1}^{(5)}$	$(CLR^3LRLR)L$	-1.962932074...
$M_{7,5}^{(33)}$	$(CLR^3LR)LRL^3$	-1.963081314...
$M_{5,5}^{(9)}$	$(CLR^3)LRLRL$	-1.963333822...
$M_{7,3}^{(10)}$	$(CLR^3LR)LRL$	-1.963581143...
$M_{8,4}^{(21)}$	$(CLR^3LRL)RL^2R$	-1.963880141...
$M_{8,4}^{(22)}$	$(CLR^3LRL)R^2LR$	-1.964143018...
$M_{6,3}^{(5)}$	$(CLR^3L)RLR$	-1.964364690...
$M_{3,5}^{(5)}$	$(CLR)R^2LRL$	-1.964783396...
$M_{7,5}^{(34)}$	$(CLR^3LR)LR^2L^2$	-1.964929421...
$M_{10,1}^{(15)}$	$(CLR^3LRLR^2)L$	-1.965160723...
$M_{7,4}^{(12)}$	$(CLR^3LR)LR^2L$	-1.965722469...
$M_{6,4}^{(9)}$	$(CLR^3L)RLR^2$	-1.965908012...
$M_{7,4}^{(13)}$	$(CLR^3L^2)LR^3$	-1.967656342...
$M_{6,4}^{(10)}$	$(CLR^3L)L^2R^2$	-1.967841749...
$M_{10,1}^{(16)}$	$(CLR^3L^3R^2)L$	-1.968372012...
$M_{3,5}^{(6)}$	$(CLR)R^2L^3$	-1.968587047...
$M_{7,5}^{(35)}$	$(CLR^3L^2)LR^2LR$	-1.968724764...
$M_{7,3}^{(11)}$	$(CLR^3L^2)LR^2$	-1.969106425...
$M_{8,4}^{(23)}$	$(CLR^3L^3)R^2LR$	-1.969299479...
$M_{8,4}^{(24)}$	$(CLR^3L^3)RL^2R$	-1.969562101...
$M_{6,3}^{(6)}$	$(CLR^3L)L^2R$	-1.969819906...

$M_{7,5}^{(36)}$	$(CLR^3L^2)LRL^2R$	-1.970035102...
$M_{5,5}^{(10)}$	$(CLR^3)L^3RL$	-1.970256767...
$M_{9,1}^{(6)}$	$(CLR^3L^3R)L$	-1.970384897...
$M_{4,4}^{(4)}$	$(CLR^2)RL^3$	-1.970613180...
$M_{8,5}^{(59)}$	$(CLR^3L^3)RL^3R$	-1.970743558...
$M_{8,5}^{(60)}$	$(CLR^3L^3)RLRLR$	-1.970980720...
$M_{7,2}^{(7)}$	$(CLR^3L^2)LR$	-1.971136027...
$M_{11,1}^{(21)}$	$(CLR^3L^3RLR)L$	-1.971327103...
$M_{6,5}^{(19)}$	$(CLR^3L)L^2RLR$	-1.971414513...
$M_{8,3}^{(17)}$	$(CLR^3L^3)RLR$	-1.971642273...
$M_{7,5}^{(37)}$	$(CLR^3L^2)LRLR^2$	-1.971732822...
$M_{8,4}^{(25)}$	$(CLR^3L^3)RLR^2$	-1.971976560...
$M_{8,4}^{(26)}$	$(CLR^3L^3)L^2R^2$	-1.972466061...
$M_{7,5}^{(38)}$	$(CLR^3L^2)L^3R^2$	-1.972630910...
$M_{8,3}^{(18)}$	$(CLR^3L^3)L^2R$	-1.972920268...
$M_{6,5}^{(20)}$	$(CLR^3L)L^4R$	-1.973024130...
$M_{11,1}^{(22)}$	$(CLR^3L^5R)L$	-1.973054717...
$M_{7,4}^{(14)}$	$(CLR^3L^2)L^3R$	-1.973109484...
$M_{8,5}^{(61)}$	$(CLR^3L^3)L^2RLR$	-1.973303813...
$M_{8,5}^{(62)}$	$(CLR^3L^3)L^4R$	-1.973702891...
$M_{5,1}^{(1)}$	$(CLR^3)L$	-1.973932044...
$M_{8,4}^{(27)}$	$(CLR^3L^3)L^3R$	-1.974340866...
$M_{7,5}^{(39)}$	$(CLR^3L^2)L^4R$	-1.974382480...
$M_{8,5}^{(63)}$	$(CLR^3L^3)L^3R^2$	-1.974572415...
$M_{8,5}^{(64)}$	$(CLR^3L^3)LRLR^2$	-1.975004337...
$M_{7,5}^{(40)}$	$(CLR^3L^2)L^2RLR$	-1.975151562...
$M_{8,2}^{(9)}$	$(CLR^3L^3)LR$	-1.975279256...
$M_{6,4}^{(11)}$	$(CLR^3L)L^3R$	-1.975517119...
$M_{10,1}^{(17)}$	$(CLR^3L^4R)L$	-1.975618967...
$M_{4,5}^{(9)}$	$(CLR^2)RL^4$	-1.975675555...
$M_{8,5}^{(65)}$	$(CLR^3L^3)LRL^2R$	-1.975773834...
$M_{7,3}^{(12)}$	$(CLR^3L^2)L^2R$	-1.975866858...
$M_{8,3}^{(19)}$	$(CLR^3L^3)LR^2$	-1.976177591...
$M_{8,5}^{(66)}$	$(CLR^3L^3)LR^2LR$	-1.976334515...
$M_{6,5}^{(21)}$	$(CLR^3L)L^3R^2$	-1.976392500...
$M_{11,1}^{(23)}$	$(CLR^3L^4R^2)L$	-1.976477895...
$M_{7,4}^{(15)}$	$(CLR^3L^2)L^2R^2$	-1.976676555...
$M_{8,4}^{(28)}$	$(CLR^3L^3)LR^3$	-1.976752066...
$M_{7,4}^{(16)}$	$(CLR^3L^2)RLR^2$	-1.977608319...

$M_{8,4}^{(29)}$	$(CLR^3L^2R)LR^2L$	-1.977683121...
$M_{11,1}^{(24)}$	$(CLR^3L^2RLR^2)L$	-1.977875753...
$M_{8,5}^{(67)}$	$(CLR^3L^2R)LR^2L^2$	-1.977957986...
$M_{6,5}^{(22)}$	$(CLR^3L)LRLR^2$	-1.978014109...
$M_{7,3}^{(13)}$	$(CLR^3L^2)RLR$	-1.978163854...
$M_{8,3}^{(20)}$	$(CLR^3L^2R)LRL$	-1.978459727...
$M_{4,5}^{(10)}$	$(CLR^2)RL^2RL$	-1.978546564...
$M_{8,5}^{(68)}$	$(CLR^3L^2R)LRL^3$	-1.978638573...
$M_{10,1}^{(18)}$	$(CLR^3L^2RLR)L$	-1.978691052...
$M_{8,4}^{(30)}$	$(CLR^3L^2R)LRL^2$	-1.978785282...
$M_{6,2}^{(3)}$	$(CLR^3L)LR$	-1.979004429...
$M_{8,5}^{(69)}$	$(CLR^3L^2R)LRLRL$	-1.979120349...
$M_{7,5}^{(41)}$	$(CLR^3L^2)RLRLR$	-1.979252567...
$M_{7,5}^{(42)}$	$(CLR^3L^2)RL^3R$	-1.979647813...
$M_{8,5}^{(70)}$	$(CLR^3L^2R)L^3RL$	-1.979814060...
$M_{5,4}^{(5)}$	$(CLR^3)L^2RL$	-1.979850178...
$M_{8,1}^{(7)}$	$(CLR^3L^2R)L$	-1.980205587...
$M_{5,5}^{(11)}$	$(CLR^3)L^2RL^2$	-1.980401188...
$M_{6,5}^{(23)}$	$(CLR^3L)LRL^2R$	-1.980741748...
$M_{8,4}^{(31)}$	$(CLR^3L^2R)L^2RL$	-1.980902792...
$M_{11,1}^{(25)}$	$(CLR^3L^2RL^2R)L$	-1.980947632...
$M_{8,5}^{(71)}$	$(CLR^3L^2R)L^2RL^2$	-1.980972494...
$M_{4,3}^{(2)}$	$(CLR^2)RL^2$	-1.981056903...
$M_{8,5}^{(72)}$	$(CLR^3L^2R)L^2R^2L$	-1.981290948...
$M_{7,4}^{(17)}$	$(CLR^3L^2)RL^2R$	-1.981413733...
$M_{7,4}^{(18)}$	$(CLR^3L^2)R^2LR$	-1.981864084...
$M_{8,5}^{(73)}$	$(CLR^3L^2R)RLR^2L$	-1.982045572...
$M_{6,3}^{(7)}$	$(CLR^3L)LR^2$	-1.982113357...
$M_{8,5}^{(74)}$	$(CLR^3L^2R)RLRL^2$	-1.982290623...
$M_{11,1}^{(26)}$	$(CLR^3L^2R^2LR)L$	-1.982357780...
$M_{8,2}^{(10)}$	$(CLR^3L^2R)RL$	-1.982504428...
$M_{6,5}^{(24)}$	$(CLR^3L)LR^2LR$	-1.982620760...
$M_{5,5}^{(12)}$	$(CLR^3)L^2R^2L$	-1.982811551...
$M_{8,4}^{(32)}$	$(CLR^3L^2R)RL^3$	-1.982908031...
$M_{9,1}^{(7)}$	$(CLR^3L^2R^2)L$	-1.983079962...
$M_{8,5}^{(75)}$	$(CLR^3L^2R)RL^4$	-1.983175336...
$M_{8,5}^{(76)}$	$(CLR^3L^2R)RL^2RL$	-1.983343981...
$M_{8,3}^{(21)}$	$(CLR^3L^2R)RL^2$	-1.983501167...
$M_{3,4}^{(3)}$	$(CLR)R^2L^2$	-1.983681352...

$M_{6,4}^{(12)}$	$(CLR^3L)LR^3$	-1.983923227...
$M_{8,3}^{(22)}$	$(CLR^3L^2R)R^2L$	-1.984055854...
$M_{8,5}^{(77)}$	$(CLR^3L^2R)R^2LRL$	-1.984341240..
$M_{8,5}^{(78)}$	$(CLR^3L^2R)R^2L^3$	-1.984456200...
$M_{10,1}^{(19)}$	$(CLR^3L^2R^3)L$	-1.984626710...
$M_{8,4}^{(33)}$	$(CLR^3L^2R)R^2L^2$	-1.985129835...
$M_{6,5}^{(25)}$	$(CLR^4)LR^3L$	-1.985540378...
$M_{8,4}^{(34)}$	$(CLR^4LR)R^2L^2$	-1.985774076...
$M_{10,1}^{(20)}$	$(CLR^4LR^3)L$	-1.986255836...
$M_{8,5}^{(79)}$	$(CLR^4LR)R^2L^3$	-1.986415068...
$M_{8,5}^{(80)}$	$(CLR^4LR)R^2LRL$	-1.986522408...
$M_{8,3}^{(23)}$	$(CLR^4LR)R^2L$	-1.986782229...
$M_{3,4}^{(4)}$	$(CLR)R^3L$	-1.986899620...
$M_{7,5}^{(43)}$	$(CLR^4L)R^3LR$	-1.986985605...
$M_{7,5}^{(44)}$	$(CLR^4L)R^2L^2R$	-1.987027244...
$M_{6,4}^{(13)}$	$(CLR^4)LR^2L$	-1.987122704...
$M_{8,3}^{(24)}$	$(CLR^4LR)RL^2$	-1.987278465...
$M_{8,5}^{(81)}$	$(CLR^4LR)RL^2RL$	-1.987413661...
$M_{8,5}^{(82)}$	$(CLR^4LR)RL^4$	-1.987558619...
$M_{9,1}^{(8)}$	$(CLR^4LR^2)L$	-1.987639168...
$M_{8,4}^{(35)}$	$(CLR^4LR)RL^3$	-1.987783334...
$M_{6,5}^{(26)}$	$(CLR^4)LR^2L^2$	-1.987862739...
$M_{5,5}^{(13)}$	$(CLR^3)RLR^2L$	-1.988023275...
$M_{8,2}^{(11)}$	$(CLR^4LR)RL$	-1.988117004...
$M_{11,1}^{(27)}$	$(CLR^4LR^2LR)L$	-1.988234989...
$M_{8,5}^{(83)}$	$(CLR^4LR)RLRL^2$	-1.988288484...
$M_{4,3}^{(3)}$	$(CLR^2)R^2L$	-1.988429001...
$M_{8,5}^{(84)}$	$(CLR^4LR)RLR^2L$	-1.988480854...
$M_{7,4}^{(19)}$	$(CLR^4L)R^2LR$	-1.988617520...
$M_{7,5}^{(45)}$	$(CLR^4L)R^2LR^2$	-1.988746669...
$M_{7,5}^{(46)}$	$(CLR^4L)RL^2R^2$	-1.988847022...
$M_{7,4}^{(20)}$	$(CLR^4L)RL^2R$	-1.988993723...
$M_{8,5}^{(85)}$	$(CLR^4LR)L^2R^2L$	-1.989080666...
$M_{6,3}^{(8)}$	$(CLR^4)LRL$	-1.989253235...
$M_{8,5}^{(86)}$	$(CLR^4LR)L^2RL^2$	-1.989314446...
$M_{11,1}^{(28)}$	$(CLR^4LRL^2R)L$	-1.989332338...
$M_{8,4}^{(36)}$	$(CLR^4LR)L^2RL$	-1.989364698...
$M_{5,5}^{(14)}$	$(CLR^3)RLRL^2$	-1.989480847...
$M_{6,5}^{(27)}$	$(CLR^4)LRL^3$	-1.989727200...
$M_{8,1}^{(8)}$	$(CLR^4LR)L$	-1.989863772...
$M_{6,4}^{(14)}$	$(CLR^4)LRL^2$	-1.990108623...
$M_{8,5}^{(87)}$	$(CLR^4LR)L^3RL$	-1.990132632...
$M_{7,5}^{(47)}$	$(CLR^4L)RL^3R$	-1.990242882...
$M_{7,5}^{(48)}$	$(CLR^4L)RLRLR$	-1.990517990...
$M_{8,5}^{(88)}$	$(CLR^4LR)LRLRL$	-1.990602172...
$M_{5,2}^{(3)}$	$(CLR^3)RL$	-1.990675930...
$M_{8,4}^{(37)}$	$(CLR^4LR)LRL^2$	-1.990816222...
$M_{10,1}^{(21)}$	$(CLR^4LRLR)L$	-1.990874996...
$M_{8,5}^{(89)}$	$(CLR^4LR)LRL^3$	-1.990907312...
$M_{6,5}^{(28)}$	$(CLR^4)LRLRL$	-1.990964964...
$M_{8,3}^{(25)}$	$(CLR^4LR)LRL$	-1.991017140...
$M_{7,3}^{(14)}$	$(CLR^4L)RLR$	-1.991201012...
$M_{4,5}^{(11)}$	$(CLR^2)R^2LRL$	-1.991286857...
$M_{8,5}^{(90)}$	$(CLR^4LR)LR^2L^2$	-1.991320635...
$M_{11,1}^{(29)}$	$(CLR^4LRLR^2)L$	-1.991366193...
$M_{8,4}^{(38)}$	$(CLR^4LR)LR^2L$	-1.991467616...
$M_{7,4}^{(21)}$	$(CLR^4L)RLR^2$	-1.991511953...
$M_{8,5}^{(91)}$	$(CLR^4LR)LR^3L$	-1.991694804...
$M_{7,5}^{(49)}$	$(CLR^4L)RLR^3$	-1.991705659...
$M_{8,5}^{(92)}$	$(CLR^4L^2)LR^4$	-1.991923863...
$M_{7,5}^{(50)}$	$(CLR^4L)L^2R^3$	-1.991934748...
$M_{8,4}^{(39)}$	$(CLR^4L^2)LR^3$	-1.992112419...
$M_{7,4}^{(22)}$	$(CLR^4L)L^2R^2$	-1.992155187...
$M_{11,1}^{(30)}$	$(CLR^4L^3R^2)L$	-1.992251383...
$M_{4,5}^{(12)}$	$(CLR^2)R^2L^3$	-1.992294178...
$M_{8,5}^{(93)}$	$(CLR^4L^2)LR^2LR$	-1.992325896...
$M_{8,3}^{(26)}$	$(CLR^4L^2)LR^2$	-1.992405486...
$M_{7,3}^{(15)}$	$(CLR^4L)L^2R$	-1.992573913...
$M_{8,5}^{(94)}$	$(CLR^4L^2)LRL^2R$	-1.992620715...
$M_{6,5}^{(29)}$	$(CLR^4)L^3RL$	-1.992672290...
$M_{10,1}^{(22)}$	$(CLR^4L^3R)L$	-1.992700939...
$M_{5,4}^{(6)}$	$(CLR^3)RL^3$	-1.992752788...
$M_{8,2}^{(12)}$	$(CLR^4L^2)LR$	-1.992875274...
$M_{7,5}^{(51)}$	$(CLR^4L)L^2RLR$	-1.992938582...
$M_{8,5}^{(95)}$	$(CLR^4L^2)LRLR^2$	-1.993009975...
$M_{8,5}^{(96)}$	$(CLR^4L^2)L^3R^2$	-1.993242042...
$M_{7,5}^{(52)}$	$(CLR^4L)L^4R$	-1.993331391...
$M_{8,4}^{(40)}$	$(CLR^4L^2)L^3R$	-1.993350646...

$M_{6,1}^{(3)}$	$(CLR^4)L$	-1.993545086...
$M_{8,5}^{(97)}$	$(CLR^4L^2)L^4R$	-1.993650764...
$M_{8,5}^{(98)}$	$(CLR^4L^2)L^2RLR$	-1.993838357...
$M_{7,4}^{(23)}$	$(CLR^4L)L^3R$	-1.993924033...
$M_{11,1}^{(31)}$	$(CLR^4L^4R)L$	-1.993947591...
$M_{5,5}^{(15)}$	$(CLR^3)RL^4$	-1.993960537...
$M_{8,3}^{(27)}$	$(CLR^4L^2)L^2R$	-1.994004687...
$M_{7,5}^{(53)}$	$(CLR^4L)L^3R^2$	-1.994127667...
$M_{8,4}^{(41)}$	$(CLR^4L^2)L^2R^2$	-1.994186600...
$M_{8,5}^{(99)}$	$(CLR^4L^2)L^2R^3$	-1.994274760...
$M_{8,5}^{(100)}$	$(CLR^4L^2)RLR^3$	-1.994386749...
$M_{8,4}^{(42)}$	$(CLR^4L^2)RLR^2$	-1.994459987...
$M_{7,5}^{(54)}$	$(CLR^4L)LRLR^2$	-1.994548141...
$M_{8,3}^{(28)}$	$(CLR^4L^2)RLR$	-1.994581209...
$M_{5,5}^{(16)}$	$(CLR^3)RL^2RL$	-1.994671864...
$M_{11,1}^{(32)}$	$(CLR^4L^2RLR)L$	-1.994705773...
$M_{7,2}^{(8)}$	$(CLR^4L)LR$	-1.994779832...
$M_{8,5}^{(101)}$	$(CLR^4L^2)RLRLR$	-1.994837209...
$M_{8,5}^{(102)}$	$(CLR^4L^2)RL^3R$	-1.994937721...
$M_{6,4}^{(15)}$	$(CLR^4)L^2RL$	-1.994984558...
$M_{9,1}^{(9)}$	$(CLR^4L^2R)L$	-1.995069132...
$M_{6,5}^{(30)}$	$(CLR^4)L^2RL^2$	-1.995115325...
$M_{7,5}^{(55)}$	$(CLR^4L)LRL^2R$	-1.995198014...
$M_{5,3}^{(5)}$	$(CLR^3)RL^2$	-1.995271594...
$M_{8,4}^{(43)}$	$(CLR^4L^2)RL^2R$	-1.995352516...
$M_{8,5}^{(103)}$	$(CLR^4L^2)RL^2R^2$	-1.995391252...
$M_{8,5}^{(104)}$	$(CLR^4L^2)R^2LR^2$	-1.995444788...
$M_{8,4}^{(44)}$	$(CLR^4L^2)R^2LR$	-1.995477114...
$M_{7,3}^{(16)}$	$(CLR^4L)LR^2$	-1.995531996...
$M_{7,5}^{(56)}$	$(CLR^4L)LR^2LR$	-1.995649507...
$M_{6,5}^{(31)}$	$(CLR^4)L^2R^2L$	-1.995696285...
$M_{10,1}^{(23)}$	$(CLR^4L^2R^2)L$	-1.995757658...
$M_{4,4}^{(5)}$	$(CLR^2)R^2L^2$	-1.995891984...
$M_{8,5}^{(105)}$	$(CLR^4L^2)R^2L^2R$	-1.995910422...
$M_{8,5}^{(106)}$	$(CLR^4L^2)R^3LR$	-1.995936920...
$M_{7,4}^{(24)}$	$(CLR^4L)LR^3$	-1.995952417...
$M_{11,1}^{(33)}$	$(CLR^4L^2R^3)L$	-1.996091341...
$M_{3,5}^{(7)}$	$(CLR)R^3L^2$	-1.996170801...
$M_{7,5}^{(57)}$	$(CLR^4L)LR^4$	-1.996185311...
$M_{3,5}^{(8)}$	$(CLR)R^4L$	-1.996565340...
$M_{7,5}^{(58)}$	$(CLR^5)LR^3L$	-1.996579318...
$M_{11,1}^{(34)}$	$(CLR^5LR^3)L$	-1.996653424...
$M_{4,4}^{(6)}$	$(CLR^2)R^3L$	-1.996779278...
$M_{8,5}^{(107)}$	$(CLR^5L)R^3LR$	-1.996792780...
$M_{8,5}^{(108)}$	$(CLR^5L)R^2L^2R$	-1.996817156...
$M_{7,4}^{(25)}$	$(CLR^5)LR^2L$	-1.996833081...
$M_{10,1}^{(24)}$	$(CLR^5LR^2)L$	-1.996948539...
$M_{7,5}^{(59)}$	$(CLR^5)LR^2L^2$	-1.996999941...
$M_{6,5}^{(32)}$	$(CLR^4)RLR^2L$	-1.997038810...
$M_{5,3}^{(6)}$	$(CLR^3)R^2L$	-1.997133781...
$M_{8,4}^{(45)}$	$(CLR^5L)R^2LR$	-1.997176882...
$M_{8,5}^{(109)}$	$(CLR^5L)R^2LR^2$	-1.997201225...
$M_{8,5}^{(110)}$	$(CLR^5L)RL^2R^2$	-1.997246334...
$M_{8,4}^{(46)}$	$(CLR^5L)RL^2R$	-1.997275019...
$M_{7,3}^{(17)}$	$(CLR^5)LRL$	-1.997335699...
$M_{6,5}^{(33)}$	$(CLR^4)RLRL^2$	-1.997389967...
$M_{7,5}^{(60)}$	$(CLR^5)LRL^3$	-1.997450112...
$M_{9,1}^{(10)}$	$(CLR^5LR)L$	-1.997483031...
$M_{7,4}^{(26)}$	$(CLR^5)LRL^2$	-1.997542399...
$M_{8,5}^{(111)}$	$(CLR^5L)RL^3R$	-1.997574599...
$M_{8,5}^{(112)}$	$(CLR^5L)RLRLR$	-1.997643522...
$M_{6,2}^{(4)}$	$(CLR^4)RL$	-1.997681502...
$M_{11,1}^{(35)}$	$(CLR^5LRLR)L$	-1.997729925...
$M_{7,5}^{(61)}$	$(CLR^5)LRLRL$	-1.997751779...
$M_{8,3}^{(29)}$	$(CLR^5L)RLR$	-1.997809435...
$M_{5,5}^{(17)}$	$(CLR^3)R^2LRL$	-1.997829881...
$M_{8,4}^{(47)}$	$(CLR^5L)RLR^2$	-1.997883521...
$M_{8,5}^{(113)}$	$(CLR^5L)RLR^3$	-1.997924587...
$M_{8,5}^{(114)}$	$(CLR^5L)L^2R^3$	-1.998003646...
$M_{8,4}^{(48)}$	$(CLR^5L)L^2R^2$	-1.998051363...
$M_{5,5}^{(18)}$	$(CLR^3)R^2L^3$	-1.998084026...
$M_{8,3}^{(30)}$	$(CLR^5L)L^2R$	-1.998152286...
$M_{7,5}^{(62)}$	$(CLR^5)L^3RL$	-1.998176232...
$M_{11,1}^{(36)}$	$(CLR^5L^3R)L$	-1.998183192...
$M_{6,4}^{(16)}$	$(CLR^4)RL^3$	-1.998195834...
$M_{8,5}^{(115)}$	$(CLR^5L)L^2RLR$	-1.998241361...
$M_{8,5}^{(116)}$	$(CLR^5L)L^4R$	-1.998338803...
$M_{7,1}^{(3)}$	$(CLR^5)L$	-1.998391361...

$M_{8,4}^{(49)}$	$(CLR^5L)L^3R$	-1.998484917...
$M_{6,5}^{(34)}$	$(CLR^4)RL^4$	-1.998493849...
$M_{8,5}^{(117)}$	$(CLR^5L)L^3R^2$	-1.998534873...
$M_{8,5}^{(118)}$	$(CLR^5L)LRLR^2$	-1.998640658...
$M_{6,5}^{(35)}$	$(CLR^4)RL^2RL$	-1.998671011...
$M_{8,2}^{(13)}$	$(CLR^5L)LR$	-1.998697583...
$M_{7,4}^{(27)}$	$(CLR^5)L^2RL$	-1.998748293...
$M_{10,1}^{(25)}$	$(CLR^5L^2R)L$	-1.998769155...
$M_{7,5}^{(63)}$	$(CLR^5)L^2RL^2$	-1.998780528...
$M_{8,5}^{(119)}$	$(CLR^5L)LRL^2R$	-1.998801025...
$M_{6,3}^{(9)}$	$(CLR^4)RL^2$	-1.998819092...
$M_{8,3}^{(31)}$	$(CLR^5L)LR^2$	-1.998883883...
$M_{8,5}^{(120)}$	$(CLR^5L)LR^2LR$	-1.998912687...
$M_{7,5}^{(64)}$	$(CLR^5)L^2R^2L$	-1.998924331...
$M_{11,1}^{(37)}$	$(CLR^5L^2R^2)L$	-1.998939312...
$M_{5,4}^{(7)}$	$(CLR^3)R^2L^2$	-1.998971995...
$M_{8,4}^{(50)}$	$(CLR^5L)LR^3$	-1.998987075...
$M_{4,5}^{(13)}$	$(CLR^2)R^3L^2$	-1.999038293...
$M_{8,5}^{(121)}$	$(CLR^5L)LR^4$	-1.999041935...
$M_{4,5}^{(14)}$	$(CLR^2)R^4L$	-1.999148079...
$M_{8,5}^{(122)}$	$(CLR^6)LR^3L$	-1.999151519...
$M_{5,4}^{(8)}$	$(CLR^3)R^3L$	-1.999198304...
$M_{8,4}^{(51)}$	$(CLR^6)LR^2L$	-1.999211633...
$M_{11,1}^{(38)}$	$(CLR^6LR^2)L$	-1.999239745...
$M_{8,5}^{(123)}$	$(CLR^6)LR^2L^2$	-1.999252332...
$M_{7,5}^{(65)}$	$(CLR^5)RLR^2L$	-1.999261967...
$M_{6,3}^{(10)}$	$(CLR^4)R^2L$	-1.999285319...
$M_{8,3}^{(32)}$	$(CLR^6)LRL$	-1.999335514...
$M_{7,5}^{(66)}$	$(CLR^5)RLRL^2$	-1.999348915...
$M_{8,5}^{(124)}$	$(CLR^6)LRL^3$	-1.999363854...
$M_{10,1}^{(26)}$	$(CLR^6LR)L$	-1.999372005...
$M_{8,4}^{(52)}$	$(CLR^6)LRL^2$	-1.999386727...
$M_{7,2}^{(9)}$	$(CLR^5)RL$	-1.999421316...
$M_{8,5}^{(125)}$	$(CLR^6)LRLRL$	-1.999438755...
$M_{6,5}^{(36)}$	$(CLR^4)R^2LRL$	-1.999458126...
$M_{6,5}^{(37)}$	$(CLR^4)R^2L^3$	-1.999521765...
$M_{8,5}^{(126)}$	$(CLR^6)L^3RL$	-1.999544666...
$M_{7,4}^{(28)}$	$(CLR^5)RL^3$	-1.999549532...
$M_{8,1}^{(9)}$	$(CLR^6)L$	-1.999598237...

$M_{7,5}^{(67)}$	$(CLR^5)RL^4$	-1.999623764...
$M_{7,5}^{(68)}$	$(CLR^5)RL^2RL$	-1.999667997...
$M_{8,4}^{(53)}$	$(CLR^6)L^2RL$	-1.999687256...
$M_{11,1}^{(39)}$	$(CLR^6L^2R)L$	-1.999692452...
$M_{8,5}^{(127)}$	$(CLR^6)L^2RL^2$	-1.999695284...
$M_{7,3}^{(18)}$	$(CLR^5)RL^2$	-1.999704891...
$M_{8,5}^{(128)}$	$(CLR^6)L^2R^2L$	-1.999731129...
$M_{6,4}^{(17)}$	$(CLR^4)R^2L^2$	-1.999742972...
$M_{5,5}^{(19)}$	$(CLR^3)R^3L^2$	-1.999759352...
$M_{5,5}^{(20)}$	$(CLR^3)R^4L$	-1.999787438...
$M_{6,4}^{(18)}$	$(CLR^4)R^3L$	-1.999799810
	...	
$M_{8,5}^{(129)}$	$(CLR^6)RLR^2L$	-1.999815649...
$M_{7,3}^{(19)}$	$(CLR^5)R^2L$	-1.999821462...
$M_{8,5}^{(130)}$	$(CLR^6)RLRL^2$	-1.999837330...
$M_{11,1}^{(40)}$	$(CLR^7LR)L$	-1.999843091...
$M_{8,2}^{(14)}$	$(CLR^6)RL$	-1.999855398...
$M_{7,5}^{(69)}$	$(CLR^5)R^2LRL$	-1.999864581...
$M_{7,5}^{(70)}$	$(CLR^5)R^2L^3$	-1.999880495...
$M_{8,4}^{(54)}$	$(CLR^6)RL^3$	-1.999887425...
$M_{9,1}^{(11)}$	$(CLR^7)L$	-1.999899589...
$M_{8,5}^{(131)}$	$(CLR^6)RL^4$	-1.999905964...
$M_{8,5}^{(132)}$	$(CLR^6)RL^2RL$	-1.999917018...
$M_{8,3}^{(33)}$	$(CLR^6)RL^2$	-1.999926233...
$M_{7,4}^{(29)}$	$(CLR^5)R^2L^2$	-1.999935743...
$M_{6,5}^{(38)}$	$(CLR^4)R^3L^2$	-1.999939826...
$M_{6,5}^{(39)}$	$(CLR^4)R^4L$	-1.999946886...
$M_{7,4}^{(30)}$	$(CLR^5)R^3L$	-1.999949968...
$M_{8,3}^{(34)}$	$(CLR^6)R^2L$	-1.999955374...
$M_{8,5}^{(133)}$	$(CLR^6)R^2LRL$	-1.999966149...
$M_{8,5}^{(134)}$	$(CLR^6)R^2L^3$	-1.999970127...
$M_{10,1}^{(27)}$	$(CLR^8)L$	-1.999974899...
$M_{8,4}^{(55)}$	$(CLR^6)R^2L^2$	-1.999983936...
$M_{7,5}^{(71)}$	$(CLR^5)R^3L^2$	-1.999984956...
$M_{7,5}^{(72)}$	$(CLR^5)R^4L$	-1.999986723...
$M_{8,4}^{(56)}$	$(CLR^6)R^3L$	-1.999987493...
$M_{11,1}^{(41)}$	$(CLR^9)L$	-1.999993725..
$M_{8,5}^{(135)}$	$(CLR^6)R^3L^2$	-1.999996238...
$M_{8,5}^{(136)}$	$(CLR^6)R^4L$	-1.999996680...

$M_{2,1}^{(1)}$	(CL)R	-2.000000000...